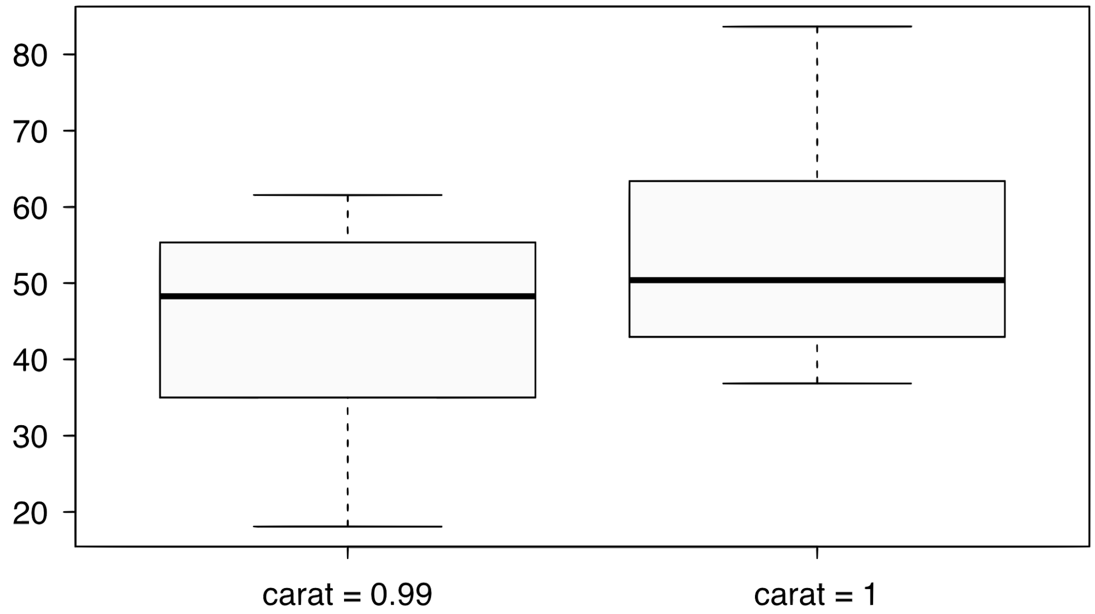
Section 7.3
Inference for a difference in two means

Stats 7 Summer Session II 2022

Diamonds

- Weights of diamonds are measured in carats
- 1 carat = 100 points, 0.99 carats = 99 points, etc.
- The difference between the size of a 0.99 carat diamond and a 1 carat diamond is undetectable to the naked human eye, but does the price of a 1 carat diamond tend to be higher than the price of a 0.99 diamond?
- We are going to test to see if there is a difference between the average prices of 0.99 and 1 carat diamonds
- In order to be able to compare equivalent units, we divide the prices of 0.99 carat diamonds by 99 and 1 carat diamonds by 100, and compare the average point prices

Data



	0.99 carat pt99	*1 carat* pt100
$\bar{x}$	44.50	53.43
s	13.32	12.22
n	23	30

Note: These data are a random sample from the diamonds data set in an R package.

Parameter and point estimate

- *Parameter of interest*: Average difference between the point prices of *all* 0.99 carat and 1 carat diamonds

$$\mu_{pt99} - \mu_{pt100}$$

- *Point estimate*: Average difference between the point prices of *sampled* 0.99 carat and 1 carat diamonds

$$\bar{x}_{pt99} - \bar{x}_{pt100}$$

Hypotheses

Which of the following is the correct set of hypotheses for testing if the average point price of 1 carat diamonds (pt100) is higher than the average point price of 0.99 carat diamonds (pt99)?

A. $H_0 : \boldsymbol{\mu}_{\text{pt99}} = \boldsymbol{\mu}_{\text{pt100}}$
   $H_A : \boldsymbol{\mu}_{\text{pt99}} \neq \boldsymbol{\mu}_{\text{pt100}}$

B. $H_0 : \boldsymbol{\mu}_{\text{pt99}} = \boldsymbol{\mu}_{\text{pt100}}$
   $H_A : \boldsymbol{\mu}_{\text{pt99}} > \boldsymbol{\mu}_{\text{pt100}}$

C. $H_0 : \boldsymbol{\mu}_{\text{pt99}} = \boldsymbol{\mu}_{\text{pt100}}$
   $H_A : \boldsymbol{\mu}_{\text{pt99}} < \boldsymbol{\mu}_{\text{pt100}}$

D. $H_0 : \bar{x}_{pt99} = \bar{x}_{pt100}$
   $H_A : \bar{x}_{pt99} < \bar{x}_{pt100}$

Conditions

- Independence within and between samples
- Normality for both samples:
  - For $n < 30$, need data to be approximately normally distributed with no clear outliers
  - For $n \geq 30$, need no extreme outliers

Conditions

Which of the following does not need to be satisfied in order to conduct this hypothesis test using theoretical methods?

A. Point price of one 0.99 carat diamond in the sample should be independent of another, and the point price of one 1 carat diamond should independent of another as well
B. Point prices of 0.99 carat and 1 carat diamonds in the sample should be independent.
C. Distributions of point prices of 0.99 and 1 carat diamonds should not be extremely skewed
D. Both sample sizes should be at least 30

Test statistics

Test statistic for inference on the difference of two small sample means

The test statistic for inference on the difference of two means where σ_1 and σ_2 are unknown is the *T* statistic.

$$T_{df} = \frac{\text{point estimate} - \text{null value}}{SE}$$

where

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad \text{and} \quad df = min(n_1 - 1, n_2 - 1)$$

Note: The calculation of the *df* is actually much more complicated. For simplicity we'll use the above formula to estimate the true df

Test statistics (cont.)

	0.99 carat pt99	*1 carat* pt100
$\bar{x}$	44.50	53.43
s	13.32	12.22
n	23	30

In context...

$$
\begin{aligned}
T &= \frac{\text{point estimate} - \text{null value}}{SE} \\
&= \frac{(44.50 - 53.43) - 0}{\sqrt{\frac{13.32^2}{23} + \frac{12.22^2}{30}}} \\
&= \frac{-8.93}{3.56} \\
&= -2.508
\end{aligned}
$$

Test statistics (cont.)

Which of the following is the correct *df* for this hypothesis test?

A. 22
B. 23
C. 30
D. 29
E. 52

$df = \min(n_{pt99}, n_{pt100}) = \min(23 - 1, 30 - 1) = \min(22, 29) = 22$

p-value

What is the p-value for this hypothesis test?

$$T = -2.508 \qquad df = 22$$

Our alternative hypothesis is that the difference is less than 0 so this is a left tail

```
> pt(-2.508, df = 22)
[1] 0.0100071
```

There's a 1.0% probability of observing data as or more extreme as what we observed, if the difference in average price per point is 0.

Synthesis

What is the conclusion of the hypothesis test? How (if at all) would this conclusion change your behavior if you went diamond shopping?

- p-value is small so reject H_0. The data provide convincing evidence to suggest that the point price of 0.99 carat diamonds is lower than the point price of 1 carat diamonds
- Maybe buy a 0.99 carat diamond? It looks like a 1 carat, but is significantly cheaper

Critical value

What is the appropriate *t** for a 90% confidence interval for the average difference between the point prices of 0.99 and 1 carat diamonds, with 22 degrees of freedom?

A. -1.72
B. 1.72
C. -2.07
D. 2.07

```
> qt((1 - 0.90) / 2, df = 22)
[1] -1.717144
```

Confidence interval

Calculate the interval, and interpret it in context

$$point\ estimate \pm ME$$

$$\begin{aligned}(\bar{x}_{pt99} - \bar{x}_{pt1}) \pm t^{*}_{df} \times SE &= (44.50 - 53.43) \pm 1.72 \times 3.56 \\ &= -8.93 \pm 6.12 \\ &= (-15.05, -2.81)\end{aligned}$$

We are 90% confident that the average point price of a 0.99 carat diamond is \$15.05 to \$2.81 lower than the average point price of a 1 carat diamond

Recap: Inference using difference of two small sample means

- If σ_1 or σ_2 is unknown, difference between the sample means follow a ***t***-distribution with $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
- Conditions:
  - Independence within and between samples
  - Normality for both samples:
    - For n < 30, need data to be approximately normally distributed with no clear outliers
    - For n ≥ 30, need no extreme outliers
- Hypothesis testing:

$$T_{df} = \frac{point\ estimate\ - null\ value}{SE}, where\ df = \min(n_1 - 1, n_2 - 1)$$

- Confidence interval:

$$point\ estimate \pm t_{df}^{*} \times SE$$